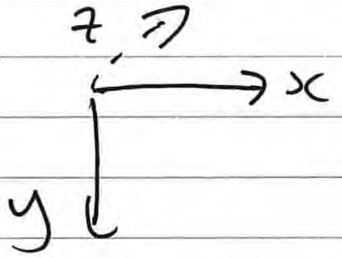
1



- A positively charged ball is dropped.
- Uniform g field acts downwards (y direction)
- Uniform B field acts into page (z direction)

$$F = m\vec{g} + q\,\vec{v}\times\vec{B}$$

Let $\vec{g} = \begin{pmatrix}0\\g\\0\end{pmatrix}$ $\quad \vec{B} = \begin{pmatrix}0\\0\\B\end{pmatrix}$

$$\therefore\ m\dot{v} = m\vec{g} + q\,\vec{v}\times\vec{B}$$

$$\dot{v} = \vec{g} + \frac{q}{m}\vec{v}\times\vec{B}$$

$$\begin{pmatrix}\dot{v}_x\\\dot{v}_y\\\dot{v}_z\end{pmatrix} = \begin{pmatrix}0\\g\\0\end{pmatrix} + \frac{q}{m}\begin{pmatrix}v_x\\v_y\\v_z\end{pmatrix}\times\begin{pmatrix}0\\0\\B\end{pmatrix}$$

$$\dot{v}_x = 0 + \frac{q}{m}(Bv_y)$$

1 $$\dot{v}_x = \frac{qB}{m}v_y$$

$$\dot{v}_y = g + \frac{q}{m}(0 - Bv_x)$$

2 $$\dot{v}_y = g - \frac{qB}{m}v_x$$

call $\frac{qB}{m}$ k

1 $$\dot{v}_x = k\,v_y$$

2 $$\dot{v}_y = g - k\,v_x$$

②

$\frac{d}{dt}$② $\Rightarrow$ $\ddot{v}_y = -k\dot{v}_x$

sub in ① $\Rightarrow$ $\ddot{v}_y = -k^2 v_y$

try $v_y = A\sin(kt) + B\cos(kt)$

initial conditions:

$t=0 \quad v_y = 0$

$\therefore \quad 0 = A\sin(0) + B\cos(0)$

$\therefore B = 0$

so $v_y = A\sin(kt)$

$t=0 \quad \dot{v}_y = g$

$\therefore \quad g = A\cos(0)$

$\therefore A = g$

so $\boxed{v_y = g\sin(kt)}$

$$v_x = \frac{g - \dot{v}_y}{k}$$

$$= \frac{g - g\cos(kt)}{k}$$

$$\boxed{v_x = \frac{g}{k}\left(1 - \cos(kt)\right)}$$

(3) $v_y = g \sin(kt)$

$$\therefore \; y_{(t)} = \int v_y \, dt$$

$$= -\frac{g}{k} \cos(kt) + c$$

when $t = 0$ $y = 0$

$$\therefore \quad 0 = -\frac{g}{k} \cos(0) + c$$

$$c = \frac{g}{k}$$

$$\therefore \; \boxed{y = \frac{g}{k}\left(1 - \cos(kt)\right)}$$

$$v_x = \frac{g}{k}\left(1 - \cos(kt)\right).$$

$$\therefore \; x_{(t)} = \frac{g}{k}\left(t - \frac{1}{k}\sin(kt) + c\right)$$

when $t = 0$ $x = 0$

so $\quad 0 = \frac{g}{k}\left(0 - 0 + c\right)$

$$\therefore \; c = 0$$

so $\boxed{x_{(t)} = \frac{g}{k}\left(t - \frac{1}{k}\sin(kt)\right)}$.

or $\boxed{x_{(t)} = \frac{g}{k^2}\left(kt - \sin(kt)\right)}$

(4) Sketch:

kt	x	y
0	0	0
$\pi/2$	$\frac{g}{k^2}\left(\frac{\pi}{2}-1\right)$	g/k
π	$\frac{g}{k^2}(\pi)$	$2g/k$
$\frac{3\pi}{2}$	$g/k^2\left(\frac{3\pi}{2}+1\right)$	g/k
2π	$\frac{g}{k^2}(2\pi)$	0